1. Let $g : \mathbb{R} \to \mathbb{R}$ be twice continuously differentiable and satisfy
\[ \int_{-\infty}^{\infty} |g''(x)| \, dx < \infty. \]
Prove that $g$ is uniformly continuous on $\mathbb{R}$.

2. Suppose \( \{f_n\}_{n=1}^{\infty} \) is a sequence of real-valued functions on $[0,1]$ such that each $f_n$ is monotone increasing on the interval $[0,1]$. If $\lim_{n\to\infty} f_n(0) = \lim_{n\to\infty} f_n(1) = A$, show that $f_n$ converges uniformly to $A$ on the entire interval $[0,1]$.

3. Compute a basis of $\mathbb{R}^3$ in which the matrix
\[
\begin{bmatrix}
1 & 3 & -2 \\
1 & -1 & 2 \\
1 & 3 & -2
\end{bmatrix}
\]
is diagonal. Your change of basis matrix will not be an orthogonal matrix; explain why no orthonormal basis can diagonalize this matrix.
4. Suppose $N$ is a complex $n \times n$ matrix. Show that $N^k = 0$ for some $k \geq 1$ if and only if there is a basis of $\mathbb{C}^n$ such that, when reexpressed in this basis, $N$ is upper-triangular with diagonal entries all equal to zero.

5. Compute the integral
\[
\int_{-\infty}^{\infty} \frac{\cos x - 1}{x^2} \, dx.
\]

6. Suppose $\tau_1, \tau_2, \tau_3, \ldots$ is a sequence of nonnegative, independent random variables each with the same probability density function $f(t) = e^{-t}$ for $t \geq 0$. Use induction to verify that for each $n$, the probability density function for
\[
S_n := \tau_1 + \cdots + \tau_n
\]
is
\[
g_n(t) = \frac{t^{n-1}}{(n-1)!} e^{-t}.
\]