1. Compute the inverse of the following matrix
\[
\begin{bmatrix}
1 & 2 & 0 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 1 & 2 \\
2 & 0 & 0 & 1
\end{bmatrix}.
\]

2. Consider the following sequence:
\[
0, \frac{1}{3}, \frac{2}{3}, 0, \frac{1}{3}, \frac{2}{3}, 4, \frac{1}{3}, \frac{2}{3}, 0, \frac{1}{3}, \frac{2}{3}, 4, 5, 6, 0, \frac{1}{9}, \ldots
\]
For a natural number \(N\), consider the first \(N\) terms of this sequence, and let \(I_N\) be the number of such terms which happen to fall within the interval \([a, b] \subset (0, 1)\). Show
\[
\lim_{N \to \infty} \frac{I_N}{N} = b - a.
\]

3. Show that
\[
\int_0^\infty \frac{\log x}{x^2 + a^2} \, dx = \frac{\pi \log a}{2a} \text{ when } a > 0.
\]

4. Find a one-to-one conformal map from the strip
\[\mathcal{S} = \{z : 0 < \text{Im} z < \pi\}\]
to the unit disk \(D_1 = \{z : |z| < 1\}\).

5. Let \(p_n\) be the probability that \(2n\) independent fair coin flips result in precisely \(n\) HEADS. Find the constants \(b\) and \(c\) such that
\[
\lim_{n \to \infty} \frac{p_n}{bn^c} = 1.
\]

6. Prove that
\[
\lim_{N \to \infty} \sum_{k=0}^{\infty} \left( 1 + \frac{k}{N} \right)^{-N} = \frac{e}{e - 1}.
\]
Any interchange of limits must be carefully justified.

7. Suppose that \(A\) is a symmetric \(3 \times 3\) matrix with positive entries, such that the sum of every row is 1. Let \(v = (1, \frac{1}{2}, \frac{1}{2})\). What is
\[
\lim_{n \to \infty} A^n v?
\]