1. Prove that the sum
\[ \sum_{n=1}^{\infty} \frac{e^{2\pi inx}}{n} \]
converges for any \( x \notin \mathbb{Z} \).

2. Suppose that \( D \) is a bounded region in \( \mathbb{R}^2 \) with a piecewise \( C^1 \)-boundary. If \( D \) is contained in a disk of radius \( R \), then prove the estimate
\[ \left| \oint_{\partial D} x \, dy \right| \leq \pi R^2. \]
In this integral \( \partial D \) is oriented as the boundary of \( D \). When is this an equality?

3. Prove: If \( f \) is a bounded increasing function defined on \( (0, 1) \), then \( f \) has at most countably many points of discontinuity.

4. Find a conformal map from the semi-circle,
\( \{ z : \text{Im} \, z > 0 \text{ and } |z| < 1 \} \),
on to the upper half plane
\( \{ z : \text{Im} \, z > 0 \} \).

5. Let
\[ f(z) = \frac{1}{1-z} + \frac{z}{2-z}. \]
Find the Laurent expansions of \( f \) valid in
(a) \( 1 < |z| < 2 \)
(b) \( 2 < |z| \).

6. Let \( A \) be the \( n \times n \) matrix:
\[ A = \begin{bmatrix} 2 & 1 & \ldots & 1 & 1 \\ 1 & 2 & \ldots & 1 & 1 \\ \vdots & \ddots & \ddots & 2 & 1 \\ 1 & \ldots & 1 & 2 \end{bmatrix} \]
(2s along the diagonal and 1s everywhere else.) Compute the determinant of \( A \).
7. Let $\mathcal{P}_n$ denote the polynomials of degree at most $n$. The first derivative defines a linear map from $\mathcal{P}_n$ to itself:

$$D : p \rightarrow \frac{dp}{dx}.$$ 

Find the matrix representation of $D$ with respect to the standard basis of monomials \{\(x^n, x^{n-1}, \ldots, x, 1\}\}. What is the Jordan canonical form of $D$? Hint: This question can be answered with very little computation.

8. A fair coin is flipped until it comes up HEADS. If this occurs on the $n^{th}$ flip, then you win $\left(\frac{3}{2}\right)^n$. What is your expected winnings?